Virtual Reality and Fractals: An exploration of immersive and emerging technologies; A case for utilization in entertainment and art installations.

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Abstract

The rapidly changing entertainment industry is teeming with innovative new technologies like Virtual Reality, Augmented Reality, Virtual Production and Artificial intelligence. The aspect of Virtual Reality as a wearable, immersive tool, having incalculable potential to give artists and game developers unlimited ways to express themselves creatively inside a 3D space, is dimension breaking. These contemporary instruments allow us to explore complex and innovative dimensional frameworks like fractals, within and possibly without, a three-dimensional environment. Game designers and programmers, in collaboration with visual artists, will take the production of immersive, interactive experiences to the next level for the end user. This paper explores how fractals and immersive game mechanics, which include physics and hand motions, will push forward the progress of the experience in which the user participates.

Acknowledgements

I would like to acknowledge Ana Tudor and Despoina Zachariadou for guiding me through the focus of this work. I would like to thank Zhan Gurskis and Herman Ho for technical support. I would like to thank Billy Beeson, James Grey, Chloe Elford, Jason Grubb, and Margaret Pushkar for their helpful discussions and play testing. I would like to thank Nicole Bullick and Bill Descher for revising my thesis. I acknowledge that my success is built upon the foundation created by others and am so grateful for all who contributed to this effort.

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Introduction

The importance of interaction and visualization in a three dimensional space cannot be overstated. Some people remember experiences through smell. Others, by recalling the sound of the memorable encounter. As for me, there is nothing more poignant than the tactility and cooperation that is provided with immersive mediums such as Virtual Reality. Virtual Reality (here-to-fore referred to as VR) is appreciated because of the interaction and responsiveness to the simulation of the sense of touch (Merriam-webster.com, 2022). The unexplored potential of VR experiences to give content creators and end users alike, a sensual cacophony of limitless exploratory options, was never more plausible than it is today.

The technology of VR allows users to experience computer generated environments and interact with objects within them as if the objects were real. VR has applications in education, training, and entertainment, which will allow realistic and intricate virtual environments for the end user. VR technology has the potential to improve learning and education by providing a highly immersive and engaging learning experience. With VR, students can explore new places, conduct experiments, and learn complex concepts in a fun and interactive way. Unlike traditional methods of teaching, which can often be dry and unengaging, VR allows students to fully engage with the content and learn in a way that feels more natural and intuitive. This can help students retain information more effectively and improve their overall understanding of the subject matter.

A real-world example would be a flight training simulator. A pilot can have realistic interaction with complex images in that virtual world without ever stepping foot on a plane.
In my project, I have my own version of a flight training simulator. I chose VR to create an experience that will teach users about fractals through gamification and immersive spaces.With gamification and design, I created an exploratory experience of fractals that aims to promote an appealing show. Fractals are complex self-similar geometric patterns found in nature that can be generated by mathematical algorithms. Fractal patterns are often used in computer graphics to make them more interesting. Using fractal patterns in VR allows for an almost infinite level of detail, which makes them useful for realistic environs. They can be quickly generated, so the real-time rendering is easy and allows for a wide range of creative possibilities; thereby creating

a unique and engaging VR experience that would not necessarily be possible with other techniques.

My project showcases fractals and fractal dimensionality. While working with these infinite shapes, artists and developers can use game mechanics unique to VR, incorporate hand interactions, and utilize the type of physics simulations impossible to encounter within a traditional desktop or gaming console. Throughout the experience the user will confront Chaos Theory, the science of surprises of the nonlinear and the unpredictable (Fractalfoundation.org, 2018). Users are introduced to the beauty of nature and geometric fractals while exploring the definitions provided to help comprehend these essential concepts, which will help them to properly make sense of Fractal Geometry and Fractals such as The Mandelbrot Set, Koch Snowflake, Pythagoras Tree, and The Menger Sponge.

Chapter 1. Literature Review, Theoretical Background

See Appendix A for Literature Sources

Fractals in VR

“Fractals' popularity are growing in the media.” (Ebb et al. 2017; Hutchins et al. 2015)
These complex geometrical shapes are intriguing for artists' inspiration and creation because they have repeating and emerging features that are visually complex. Virtual reality is beneficial when displaying these because of the six degrees of freedom that a user has when viewing content. Artists can be inspired with this new tool to better comprehend graphics and rendering for content creation. Tools such as ray marching and signed distance fields are likely to improve fractal rendering in VR experiences.

Rendering fractals in Virtual Reality is unique because it gives the user a new perspective on these shapes and scales. 3D spaces such as Virtual Reality give us a tool that can symbolize infinite spaces and shapes. This is achievable by using techniques like signed distance functions: “These are mathematical functions that take a point in space and tell you how far that point is from a surface,) (Saam et al., 2018) “ or advanced rendering techniques such as stereo reprojection and ray marching. “Raymarching - It’s an interesting technique that can be used to generate fully procedural environments entirely from a single fragment shader.” (Saam et al., 2018)

Due to the abstract and chaotic nature of fractals, the main design challenges when developing an interactive VR explorer consist of the development of a suitable locomotion model and the discernment of how to navigate repeating patterns at vastly different scales (ACM Conferences, 2018).

Generation of 3D Fractal Images for Mandelbrot and Julia Sets, 2022

Fractal geometry is a field of mathematics that deals with the study of geometric figures that are self-similar and exhibit a repeating pattern at different scales. This field of mathematics has been applied to a wide range of natural phenomena, including the study of tree growth and behavior. One of the most famous examples of a fractal is the Mandelbrot set, which is a mathematical object that is defined by a simple equation and exhibits complex, self-similar patterns. I am fascinated by fractals being found all around us. I decided to use various fractals in my project, the Mandelbrot and Julia sets, geometric fractals and other fractals found within nature. ((1) (PDF) Generation of 3D Fractal Images for Mandelbrot and Julia Sets, 2022)

Fractal geometry is a mathematical concept that studies the properties of geometric shapes that are self-similar, meaning that they can be divided into parts that are scaled versions of the whole. “As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality.” -Albert Einstein (Fractalfoundation.org, 2018). This type of geometry is often used to describe complex and irregular shapes that cannot be represented accurately using traditional Euclidean geometry.

Fractal geometry has a wide range of applications, including computer graphics, physics, and biology. In computer graphics, fractal geometry is often used to generate complex and realistic-looking landscapes, clouds, and other natural phenomena. In physics, fractal geometry is used to describe the behavior of complex systems, such as the distribution of galaxies in the universe. In biology, fractal geometry is used to study the structure of plants and other natural objects.

Overall, fractal geometry is a fascinating and versatile mathematical concept that has many applications in various fields, including computer graphics, physics, and biology. Hence the reason that I thought fractals would be a great exploratory focus.

Fractals were present in every scene that I created, eight total portal scenes and one main hub. Two scenes use fractals as skyboxes and are visually stimulating, two scenes teach users about natural fractals, and two scenes are interactive with the player.

Educational Virtual Environments: A ten-year review of empirical research(1999–2009)

VR has the potential to revolutionize the field of education by providing a highly immersive and engaging learning experience. With VR, students can explore new places, conduct experiments, and learn complex concepts in a fun and interactive way. In this review of empirical research, I will be reviewing how VR can provide a deeper understanding of the subject matter and make the learning experience more engaging. This can help to deepen scholars' understanding and appreciation of the artwork and potentially lead to a more fulfilling and enriching experience:

The statement "Information and Communication Technologies (ICT) are considered to be one of the most powerful tools for the support of the learning process" highlights the potential benefits of using technology in education. According to Jonassen (1999) and Smeets (2005), ICT can play a critical role in enhancing the learning experience and facilitating more effective teaching and learning.

One of the main advantages of using ICT in education is that it can provide access to a wide range of learning resources and information. With the increasing availability of online resources and digital content, students can access a vast amount of information and learning materials from anywhere, at any time. This can be particularly useful for students who may not have access to traditional learning resources, such as books and libraries, and can help to level the playing field for all learners.

Additionally, ICT can also support more interactive and engaging forms of teaching and learning. With the use of technology, teachers can create dynamic and interactive learning experiences that can motivate and engage students. This can include the use of multimedia resources, such as videos and simulations, as well as collaborative tools and platforms that enable students to work together and share their knowledge and ideas. As stated in the paper

“VR can be described as a mosaic of technologies that support the creation of synthetic, highly interactive three dimensional (3D) spatial environments that represent real or non-real”

Featuring: Fractals displays low poly and photoscanned assets together, only something that could be done with art. This description shows that VR can be pedagogically exploited through its unique technological characteristics that can be compiled as follows (Mikropoulos & Bellou, 2006):

- creation of 3D spatial representations, namely virtual environments (VE)
- multisensory channels for user interaction
- immersion of the user in the VE
- intuitive interaction through natural manipulations in real time.

Chapter 2. State Of The Art

In this section I conduct case studies and review art installations that have the potential for (VR) technology to benefit by providing a highly immersive and engaging experience for viewers. With VR, viewers can be transported to a different time and place, fully immersing themselves in the artwork and experiencing it in a way that is not possible with traditional art installations.

One way that VR can benefit art installations is by allowing viewers to experience the artwork from different perspectives. I chose to show this by using a climbing interaction. I specifically chose this interaction because of the hand tracking that takes advantage of an aspect that is unique to Virtual Reality. I chose fractals because of their self-similar and repetitive characteristics. I thought this would be perfect for an exploration game. Fractals make great mazes and I thought that since the user was in VR this would really emphasize the fact that it lets you see a virtual object or environment from different angles which provides a more immersive and engaging experience than simply viewing it from one fixed perspective.

Another potential benefit of VR in art installations is that it can be used to create interactive experiences. For example, my VR installation used game mechanics to allow the user to interact with the fractal via archery and skybox adjustments. This can make the viewing experience more engaging and interactive, allowing viewers to feel more connected to the artwork.

See Appendix B for Youtube Video.

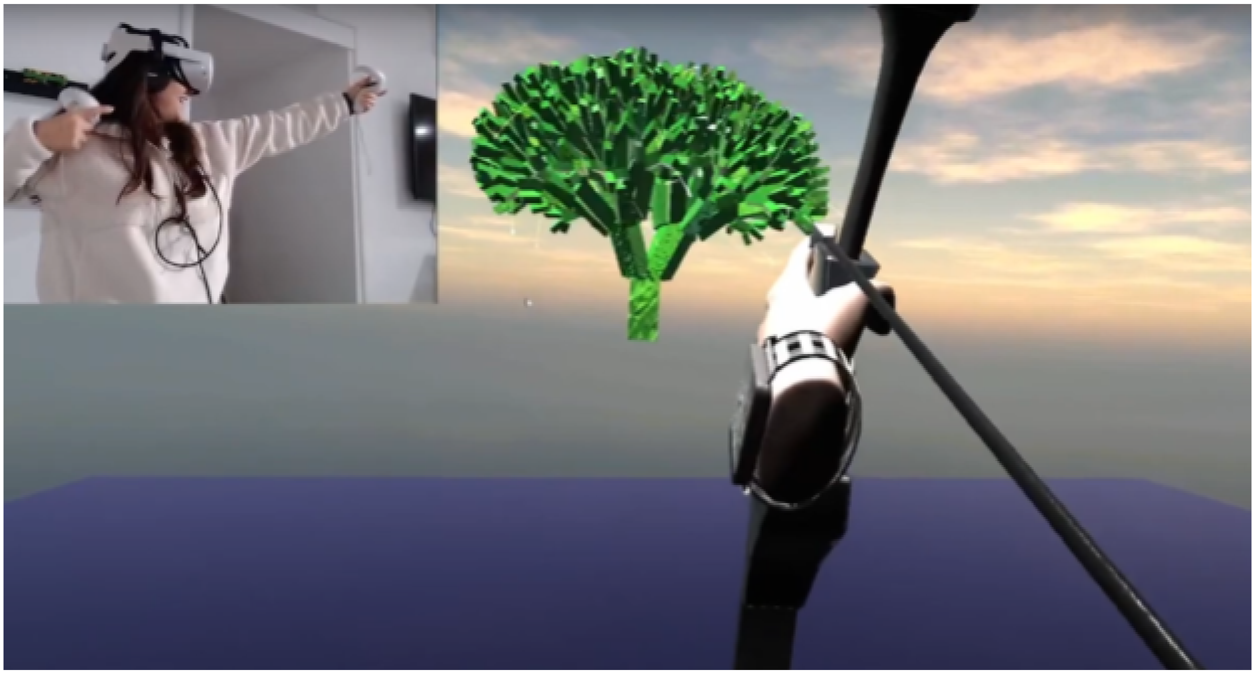


Figure 1: Despoina shooting a bow and arrow to iterate the fractal on collision.

Virtual Art Installations

Museum of Other Realities

Museum of Other Realities, is a VR gallery application that gives artists a space to create content in 3D and make immersive art. *VRChat* and *Museum of Other Realities* were the chosen applications that I explored to view Fractals inside of VR.

See Appendix C for *Museum of Other Realities* Gameplay.

Figure 2: Museum of Other Realities, Fractal World

In the Museum of other realities, I was struck by the fractal gallery. The abstract colors and varied shapes of the fractals caught my attention, and I found them to be a great source of inspiration for my own work. I spent some time studying the fractals, taking note of their unique features and exploring their intricacies. Overall, I found the experience to be both stimulating and enlightening.

VRChat

See Appendix D for *Fractal World* VRChat Gameplay and sources for VRChat Worlds.

VRChat is a platform that allows artists to create content and worlds to uploadable into their platform so people can explore their creations. This specifically is beneficial whilst in VR because users can participate in interactions that would not be possible on a desktop or console. *VRChat* is well known for the creativity of its community and that it is open source.

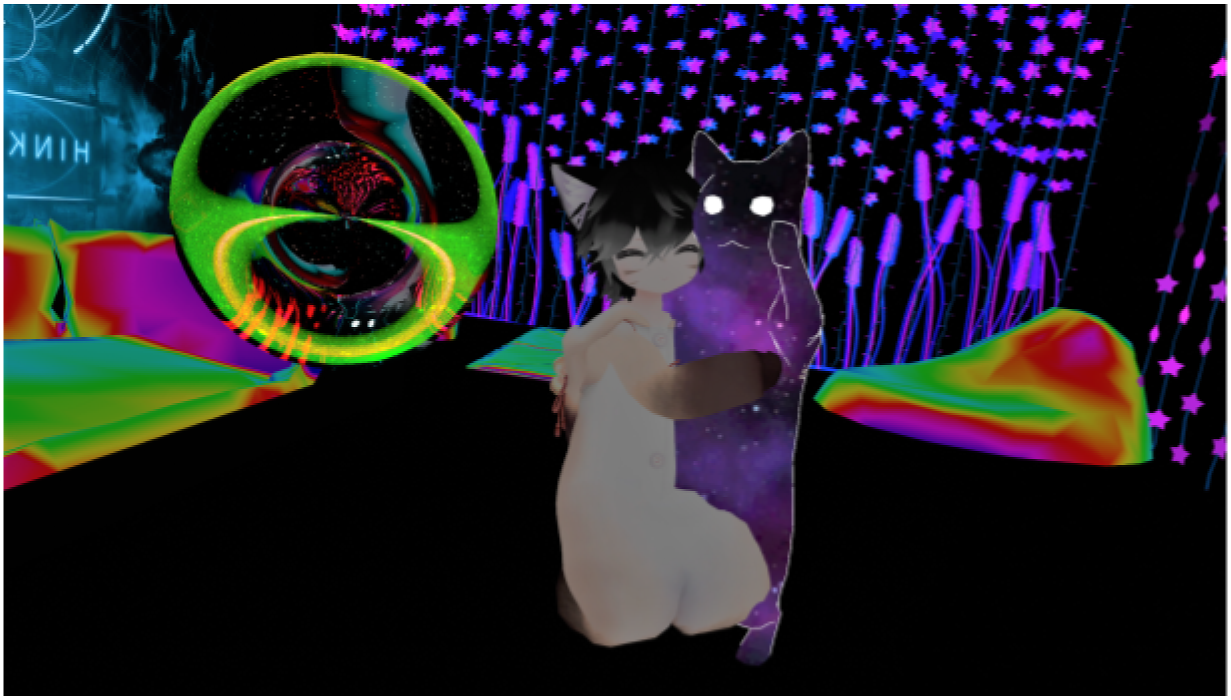


Figure 3: My colleague and I exploring 3D fractals in VRChat

This is me (Galaxy Cat) inside of fractal world with my friend (13illy). My colleague instructed me to grab this ball and look inside of it. Once I poked my head into the ball by placing it over my head, I saw a beautiful fractal.

As I picked up the fractal with my virtual avatar's hands, I was able to explore the intricacies of the fractal inside the ball. However, this experience was only possible because I had access to hand tracking technology, such as the Quest two controllers. Without this technology, interacting the fractal with a mouse and keyboard, which is common in desktop gaming, would not have been possible.

Figure 4: This VRChat world is viewed through a distortion shader

There were fractal shaders that I could explore and envision the world through different trippy and distorted shaders. I liked the idea of my entire field of view being changed so I wanted to recreate a scene that changed the skyboxes to different fractal shaders since I knew that would affect everything in the environment including the light reflections.

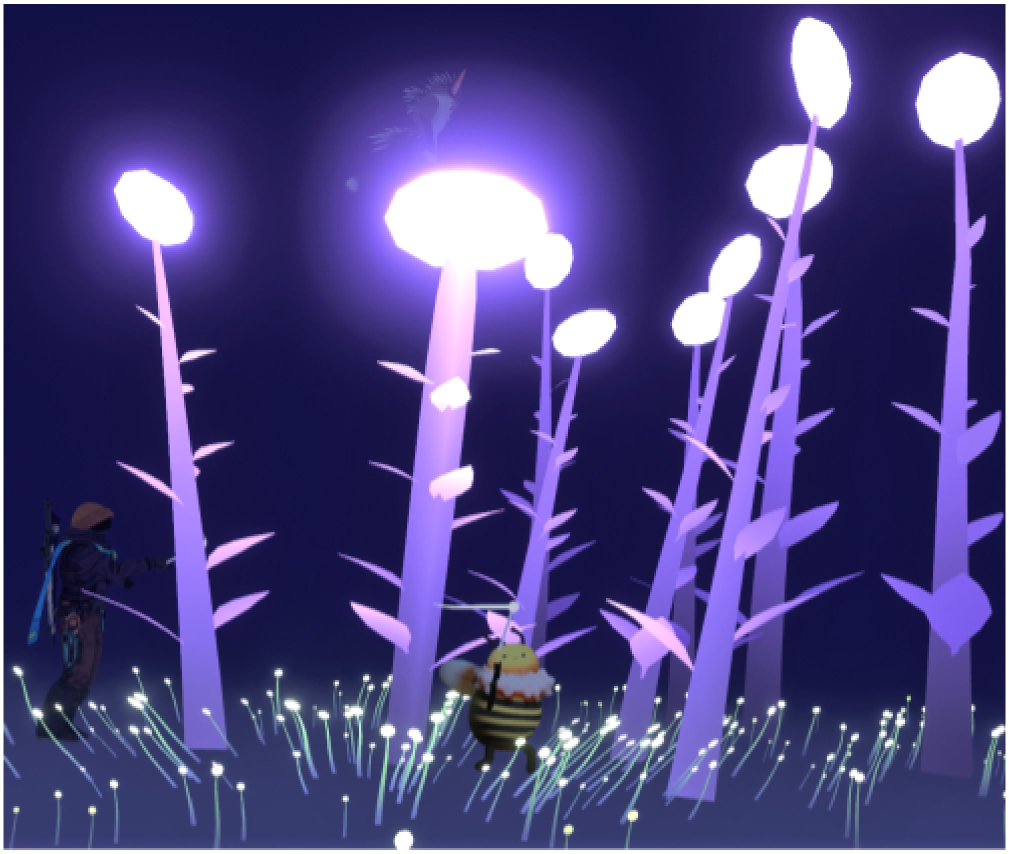


Figure 5: World Exploring in VRChat

Figure 6: This VRChat world inspired Sky Full Of Fractals

The 360 degree videos that I watched inspired my own content by showcasing the beauty and sophistication that can be achieved with this immerisve technology. As I watched the visualizations unfold around me, I was struck by the level of detail and complexity that was possible. This inspired me to try and capture the same level of beauty in my own work.

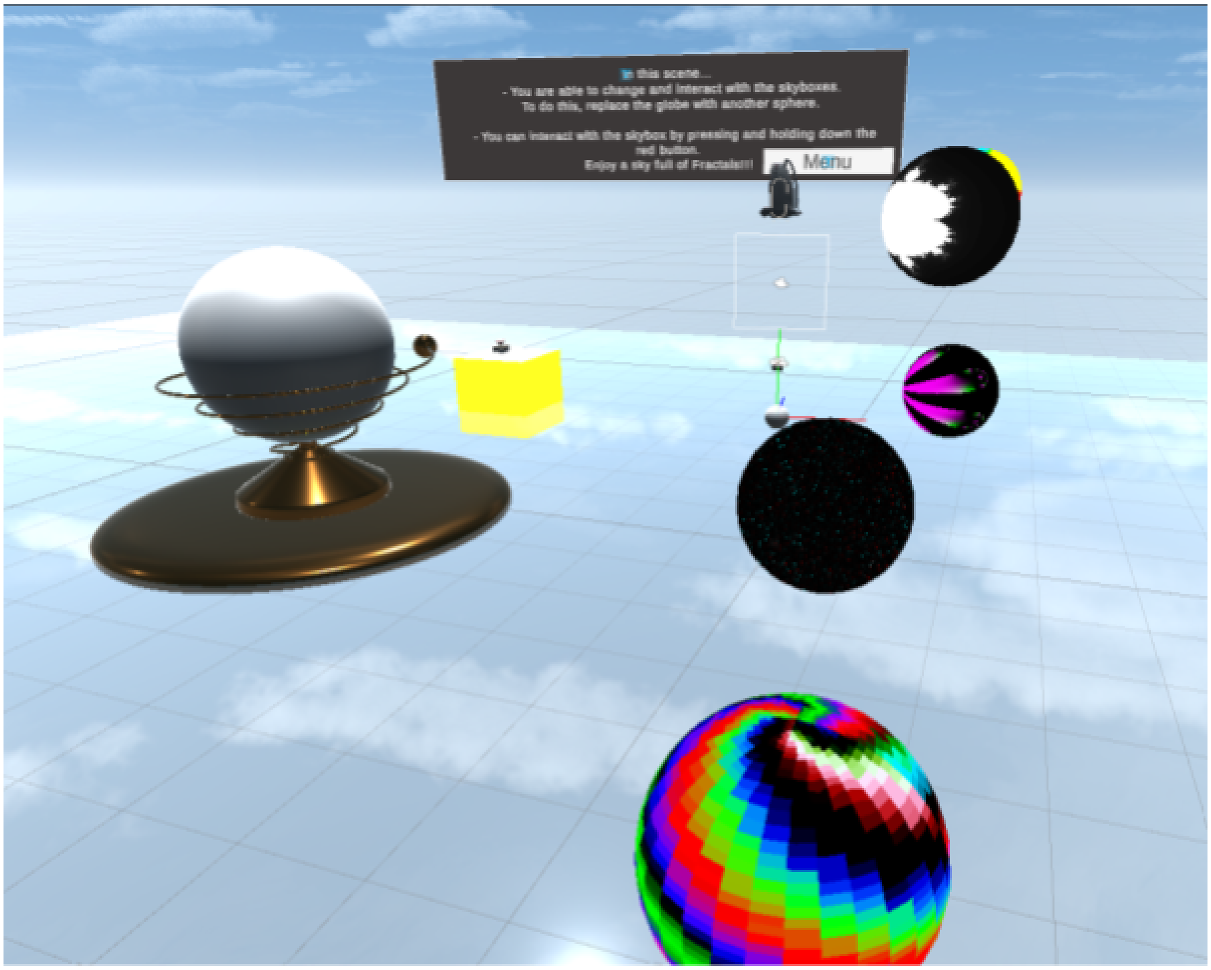


Figure 7: Sky Full of Fractals, Interactive Scene

Conway’s Game of Life

See appendix E for GIF and Display of Conway’s Game of Life.

Conway's Game of Life is a computer game that can be played repeating patterns infinitely based on the written code. Gamification and coding allows programmers to make experiences that are infinite, self-sufficient, and generated procedurally (Wikipedia Contributors, 2022). When creating my final major project, I wanted to create a game that was unique to VR and I thought that code would be an efficient way to do this. Conway’s game of life piqued my interest because Zhan showed our class this was a way to make a repetitive game generated by code. Since one of my themes was infinity, I chose to include Conway's Game of Life into my case studies.

The starting pattern, or seed, is the basis for the system. The rules are applied to every cell in the seed simultaneously, resulting in the creation of the first generation. This process, called a tick, happens instantaneously and determines which cells will live and which will die. Each subsequent generation is determined solely by the previous generation, and the rules continue to be applied repeatedly to create new generations."(Wikipedia Contributors, 2022)

VR Games

No Man's sky

Another inspiration that I encountered was *No Man's Sky* where I was inspired by the procedural generation of these worlds. This is where I began my focus on infinity and combined it with the exploratory concept that I got from *Hyperbolica*. Procedural generation is a technique used in computer programming to generate data or content algorithmically, rather than manually created. The advantage of this approach is that it can create vast amounts of unique content quickly and efficiently. *No Man's Sky*, a science fiction game set in an infinite procedurally generated universe, is one of the most prominent examples of this technology in action (Non Scalability and generating digital outer space natures in *No Man's Sky*, 2021). In a 2017 talk at the Game Developers Conference, Sean Murray of Hello Games described some of the challenges and technologies involved in creating both realistic and alien landscapes without any artistic input, using only mathematics. (Canadalorian, 2019)

One of the key challenges in procedural generation is ensuring that the generated content is believable and convincing to the player. In *No Man's Sky*, this means creating landscapes that look realistically varied and lush, while still maintaining a sense of coherence and order. To achieve this, Murray and his team used a number of different algorithms, including noise functions (which are commonly used in computer graphics to create natural-looking textures) and Perlin noise (a type of gradient noise that produces more organic results). Another challenge is creating alien landscapes that are convincingly different from anything found on Earth. This requires inventing entirely new algorithms or tweaking existing ones to produce unexpected results. For example, by changing the way Perlin noise is applied, Murray was able to create strange, otherworldly terrains with jagged peaks and valleys. By experimenting with different values and settings, it's possible to create an infinite variety of unique things.

Figure: 8 No Man's Sky, procedural generation game

My experience was positive and I enjoyed the game, especially exploring the space.I thought the game was adequate but not great in translation to Virtual Reality, as it was originally created for a console or desktop gaming setup. Because the game is so heavy the graphics are not incredible and the UI seems as if it wasn't the most efficient for a 3D interactable space. I did really enjoy the idea that this game can go on forever and that is what inspired my climbing mechanic as it started as an infinite climbing level and then I changed it to be a way of navigating through the Menger Sponge.

Developers can create spaces that are infinite with procedural generation! The user will infinitely play different experiences and can be entertained much longer than in traditionally created environments, thereby leveraging the mechanics to games and experiences, the experience remains fresh with less chance for user fatigue.

Hyperbolica

See Appendix F for *Hyperbolica* Gameplay and Interview with developer.

Hyperbolica was the initial and primary game that inspired *Featuring: Fractals.* I subsequently reviewed an interview with CodeParade who is a prominent game developer on YouTube as well as the developer of *Hyperbolica*.

Hyperbolica is a game with mind-bending world design based on Non-Euclidean curved space. It's a thoroughly quirky adventure, with a layout using a hyperbolic geometric style. *Hyperbolica* is a story-based adventure filled with exploration, mini-games and more. (Dawe, 2022)

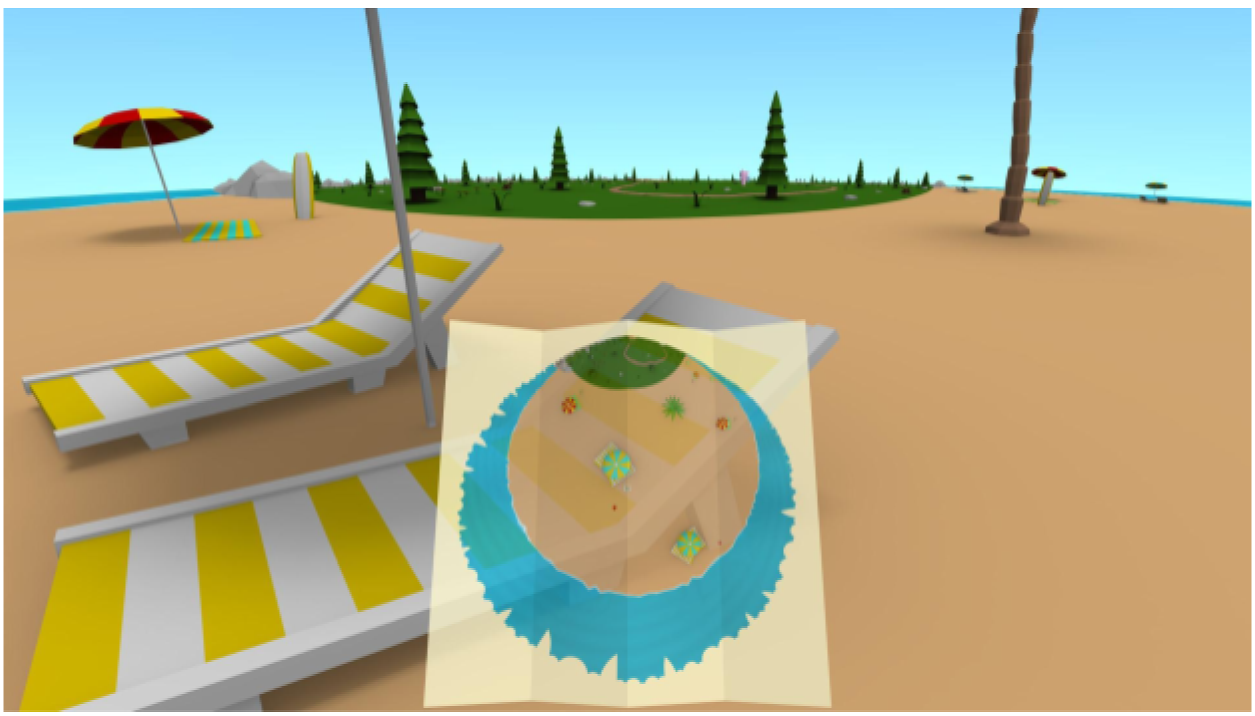


Figure 9: Hyperbolica gameplay

Hyperbolica is a first-person exploration and adventure game designed to experience what it's like to live in a non-Euclidean space. In particular, it explores hyperbolic and spherical geometries, which have very interesting gameplay consequences. While the mathematical components are intimidating (even to me), *Hyperbolica* makes it easy to discover and learn about everything through pleasing minigames and interactions so that everyone can build intuition and understand what's going on. It was available for desktop PC and VR at launch, and may have console ports in the future.

"This developer explains that this is an exploration and adventure game. *Hyperbolica* lets players feel what it's like to live in a non-Euclidean environment. It focuses on hyperbolic and spherical geometries in particular because of the fascinating gameplay implications they provide.The developer states that the mathematics used to create this game are even intimidating to himself but when launched the only other game like it was hyperrogue. His own curiosity about what it would be like to walk in or fly around a hyperbolic world was definitely the main inspiration." (Harris, 2021) (Quillbot.com, 2022)

Chapter 3. My Project

When I started this journey of choosing my thesis project, *Featuring Fractals*, I wished to create a VR experience that used gamification to explore fractals and worlds created using mathematics. I wanted this experience only to be playable using VR and not accessible for traditional gaming platforms. Virtual reality (VR) technology allows for the creation of immersive, interactive experiences that are not possible with traditional gaming. Because VR allows users to fully engage with a digital environment using their senses, it has the potential to create experiences that are more realistic and engaging than those in traditional games. For example, VR can create the sensation of being in a physical space, even if that space does not actually exist. This can allow for the creation of environments that would be impossible or impractical to create in the real world, like height sensitive expansive and futuristic worlds. Additionally, VR can create experiences that are more interactive and dynamic than those in traditional games, allowing users to fully engage with the environment and make choices that affect the outcome of the experience.

I wanted to use mathematics and physics to create a game for the user to explore. The goal of this project was to create a sandbox full of geometry, interactions, and visuals whilst educating on fractals as well. This focus was originally inspired by non-euclidean geometry. Honestly, I was just trying to make a noneuclidean portal package work. I was fascinated by the illusion this

mechanic portrayed for the user. It was magical and it is a unique illusion that can only be achieved using a game engine.
See Appendix G for Unity Non-Euclidean Portal Package.

NonEuclidean geometry is based upon one or more postulates that differ from those of Euclid, especially from the postulate that only one line may be drawn through a given point parallel to a given line. There are two main types of non-Euclidean geometries, spherical (or elliptical) and hyperbolic. They can be viewed as opposing or complimentary, depending on the aspect we consider. (Cornell.edu, 2022)"

I was playing *Hyperbolica*. I found this game interesting because of the geometry that was used to create the environment. I liked that *Hyperbolica* used many environments that were unrelated to each other. I thought that would be an exhilarating way to make a game. At the beginning of my thesis I did not have much structure on where I wanted to go with my theme or environment creation so I sent out surveys to people I knew.
See Appendix H for Survery.

I became inspired by *Hyperbolica*. As computer technologies have advanced, it seems to me that there has been a higher demand in content creation of abstract mathematical experiences through the lens of gaming.

"*Hyperbolica,* a game where you explore non-Euclidean worlds. It offers an experience you can only get from playing a video game. Even trying to envision what kinds of sights are typical in Hyperbolica is tough. There are a few other concepts that are somewhat comparable, but nothing quite like it." (Harris, 2021)

I saw this as an opportunity to show people what Virtual Reality has the ability to display in 3D immersive spaces. There is a puzzle type feeling this game possesses that was hypnotic. When playing the desktop version, it felt pretty basic. When I realized that *Hyperbolica* had a Desktop and VR version, I was keen to see how the two compared. There literally was no comparison. The VR *Hyperbolica* game exists on a completely different level. I was intrigued by the fact that game engines gave me the tools to create an experience that is impossible to do without these tools. Immediately after comparing the two adaptations, I realized VR's enhanced capability to portray non-euclidean geometries was an incredible learning tool and visually very captivating. *Hyperbolica* is appealing to me because I love world creation and designing a game with myriad puzzles and branching mini games included.

My concept spawned from fractal and noneuclidean geometric spaces. I was hooked! The idea of mathemtaically generator scenes was innovative and the space looked incredible from the vantage point of being inside of it. I became mesmerised by the idea that these worlds were created using geometry. *Hyperbolica* combined some of my favourite things: flying drones and snow skiing, all into one game. It is stunning how smoothly two different activities are able to fit so flawlessly into this one experience. Researching non-euclidean geometry which led to an interest in fractal spaces made me desire to recreate something similar.

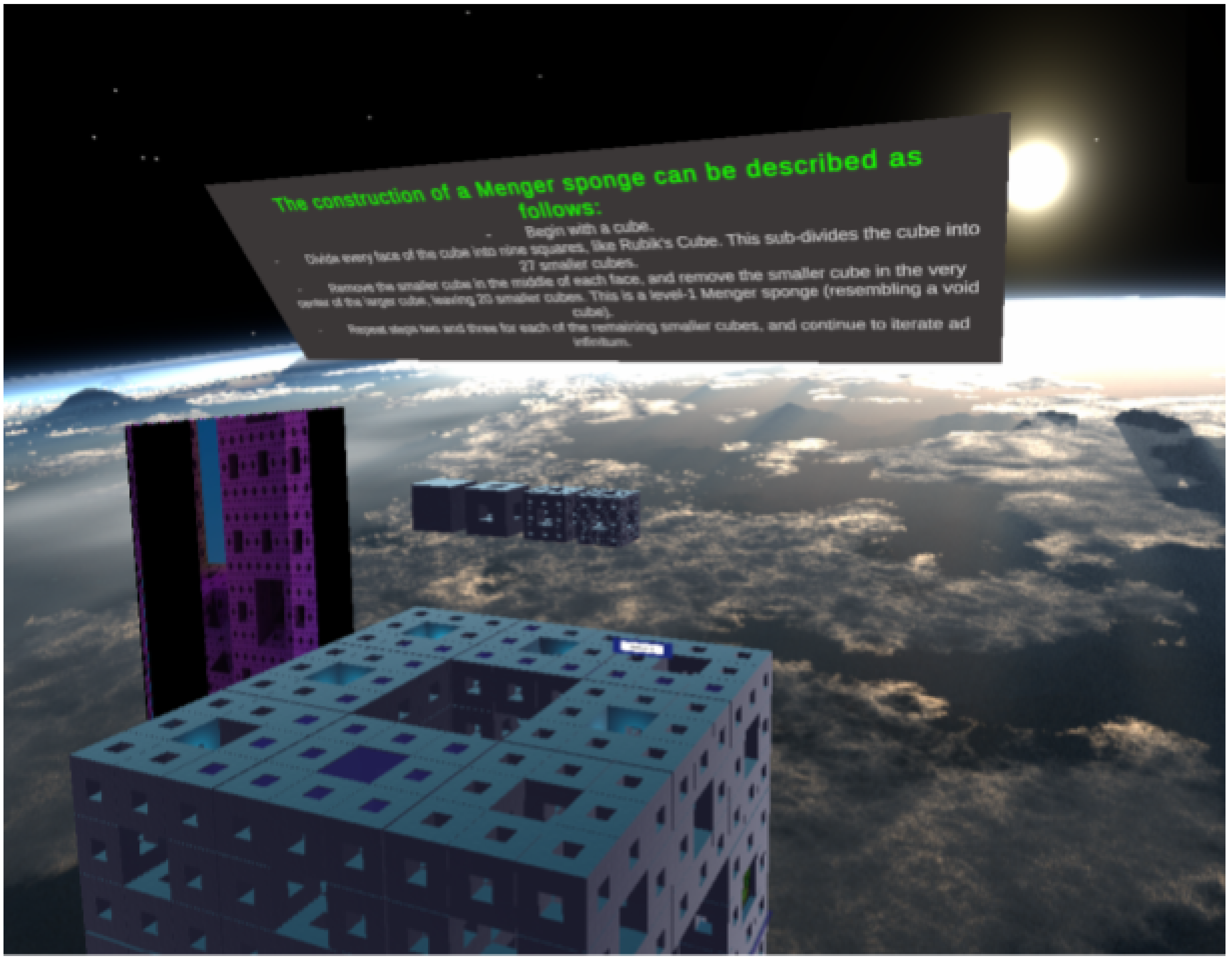


Figure 10: Menger Sponge Menu, Bird's Eye View

This is where the interest in Extended Reality or XR comes into focus. XR includes technologies such as virtual reality (VR), augmented reality (AR), and mixed reality (MR). These technologies allow users to interact with digital objects and environments in a way that feels real, providing a unique and engaging way to learn, play, and experience new things. XR technology is often used in education, gaming, entertainment, and other industries. I saw this as the perfect opportunity to begin steering my project to create a game using three tools that I wanted to incorporate into my Final Major Project: Hurricane Physics VR package, exploration of fractal geometric spaces, and mathematical concepts.

The use of Virtual Reality (VR) as a tool for exploring and learning about fractals has been the focus of my research. The results of this research demonstrate that VR is an effective way to engage with and understand fractals. One of the challenges of working with fractals is that they can be overwhelming in two-dimensional (2D) zoom representations. Therefore, I have focused on creating various VR scenes that allow users to explore and learn about fractals in a more accessible way. This work has also highlighted the potential for VR to be used as an educational tool, helping people to understand and appreciate the beauty and complexity of fractals.

The first step was to acquire a Hurricane VR kit. This toolkit uses physics interactions and supplied me with an XR Rig. I really enjoyed learning about physics in our coding classes and I wanted to challenge myself by implementing gamification into Virtual Reality technology using those types of physics. I used a Unity package called Hurricane VR Interaction Toolkit.
See Appendix I for Unity Hurricane Interaction VR Toolkit.

Physics can be used in VR to create more realistic and immersive interactions. VR physics simulations can be used to model the behavior of objects in the virtual environment. This adds a more natural and intuitive feel for the user as the objects will behave in a way that is consistent with the laws of physics. I used physics in my game with the bow and arrow to change the fractals from simple early interactions and on collision the fractals would self replicate and become more simple with each arrow hit.

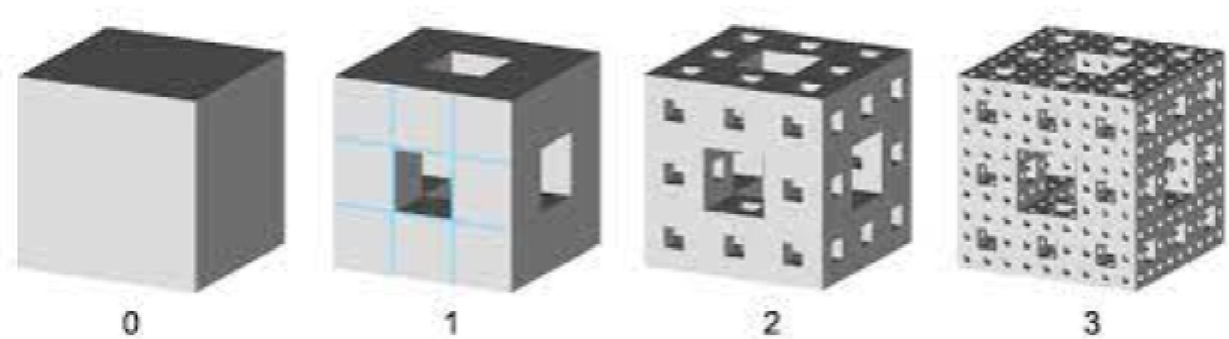


Figure 11: Menger Sponge three iterations

Overall, the use of physics in VR has enhanced the realism and immersion of virtual environments filled with 3D geometric shapes. VR allows users to interact with the virtual environment in a way that feels natural and intuitive, creating a more engaging and immersive experience.What better way to experiment with my thesis question, than to use the interactions that were making me excited, such as the climbing feature or the bow and arrow movements. The interaction kit from Hurricane VR allowed me to create a place where my users are able to explore all of the dynamics of these fractal shapes. It turns out VR is the perfect tool for the job. Using a game engine for this exploration has allowed me to work with immersive tech, tracking devices, physics interaction kits, and more, all inside one project. I wanted to show my viewers a project absolutely full of fractals and Visual immersive effects.

Working inside of a 360 space allowed me to program interactions that would be nearly impossible without this kind of technology. Taking advantage of this I created extremely immersive scenes roughly half of which are adjustments of the 360 skyboxes inside of this Virtual reality project. To me, hand tracking interactions are the biggest differing factors between XR and console or desktop gaming. I enjoy the physical activity and immersion that comes with trackable and wearable technology. I knew that I wanted to use this project to show my players that VR can open new physical and visual opportunities. I focussed on emphasizing this point by choosing my mechanics with the thought of what is immersive about VR and how we can exhibit this to people. After the exploration of mathematical equations used in the code, I realized that fractals would be a suitable fit for the content of this project.

Fractals

"A fractal is a geometric shape that is self-similar and has a fractional dimension" (p. xiii). In other words, a fractal is a geometric object that is made up of smaller versions of itself and has a dimension that is not a whole number. Fractals are often used in computer graphics and other applications because of their complex and infinitely detailed structure." (Benoit Mandelbrot, 2021)

I wanted to show how VR can be used as a tool, fractals were visually beautiful and so fascinating and I thought the end user would think so as well. I wanted to make the experience fun, interactive, visual, and engaging which also served to teach people along the way.

The fractal shapes I began with are the Menger sponge, Koch snowflake, Pythagoras tree, and the Romanesco Broccoli. I thought these would be good puzzle exploratory fractals for people to explore my portals through. I struggled to get non-euclidean portals working so I switched to study fractal geometry. I started with the Menger Sponge because it was one of the most recognizable fractals and I thought it would serve as a good space to showcase all of my portals and introduce the exploratory experience the user would be going through. You will recognize the following Menger sponge from my project:

"In mathematics, the Menger sponge (also known as the Menger cube, Menger universal curve) is a fractal curve. It was first described by Karl Menger in 1926, in his studies of the concept of topological dimension". (Wikipedia Contributors, 2022).

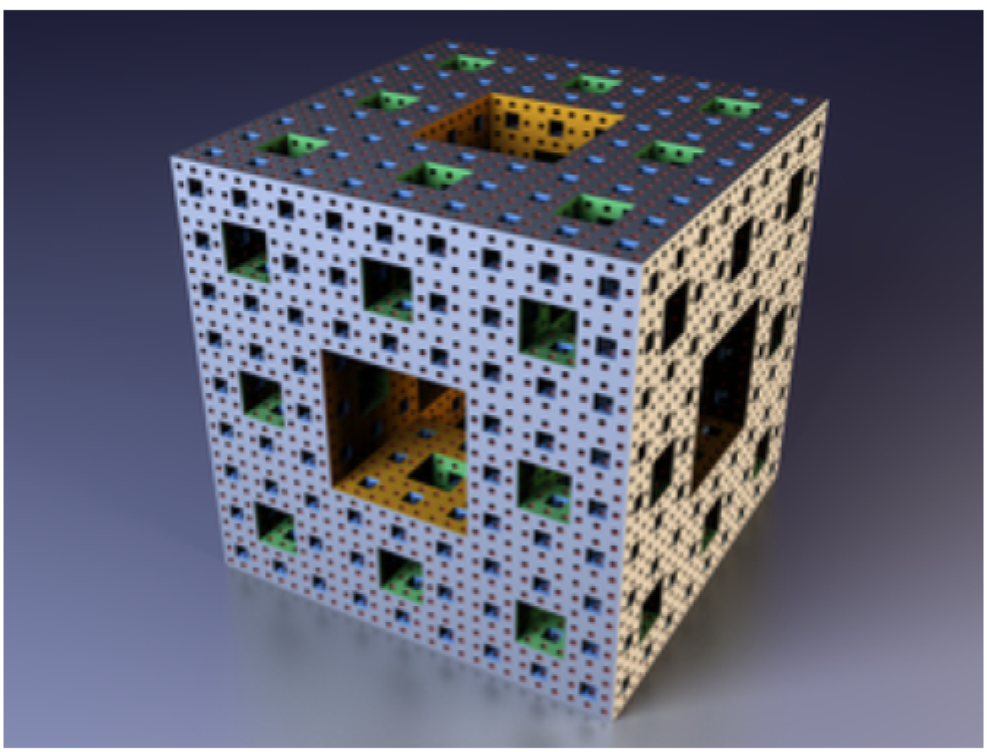


Figure 12: Menger Sponge

See Appendix J for Menger Sponge Animation

Menger Sponge

The Menger Sponge Menu I used is formed by using the Menger Sponge, a fractal named after the mathematician Karl Menger. It is created by starting with a cube and then dividing it into 27 cubes. The Menger sponge is a fractal that is named after the mathematician Karl Menger. It is created by starting with a cube and then dividing it into 27 small cubes, removing the cube in the middle and the six cubes that touch it, and then repeating this process on each of the remaining 20 cubes. This process is repeated indefinitely, resulting in a sponge-like shape with an infinitely complex structure. The Menger sponge has the interesting property that it has a finite volume but an infinite surface area. It is also a self-similar fractal, meaning that a smaller version of the sponge can be found within the larger one. The Menger sponge has been studied extensively in the field of mathematics and is also used in computer graphics and other applications. In "Featuring: Fractals" the user spawns in a Menger Sponge Menu. The user can always come back to this scene as this is the home base for all of the teleportation done to each fractal scene.

I wanted to create virtual worlds that were themselves fractals. To do this, I used another unity package called Fractal Generator to build my environments.
See Appendix K for Unity Fractal Generator.

My main hub was a Menger Sponge, which I placed in outer space to give users a sense of being out of body. The Menger Sponge itself was a maze, and users enjoyed exploring it and teleporting around. I initially considered implementing gravity controls, but ultimately decided against it because it made it complicated for the user and I liked the challenge that climbing provided for physical activity. Instead, I have considered allowing users to teleport to the second level, where they had access to four different portals. Many users were frustrated when the teleporting mechanic would not reach the second level. In the future, I will consider adding the gravity controls or other features to improve the user experience. Overall, my project created a fun and engaging environment where users could learn about and explore fractals.

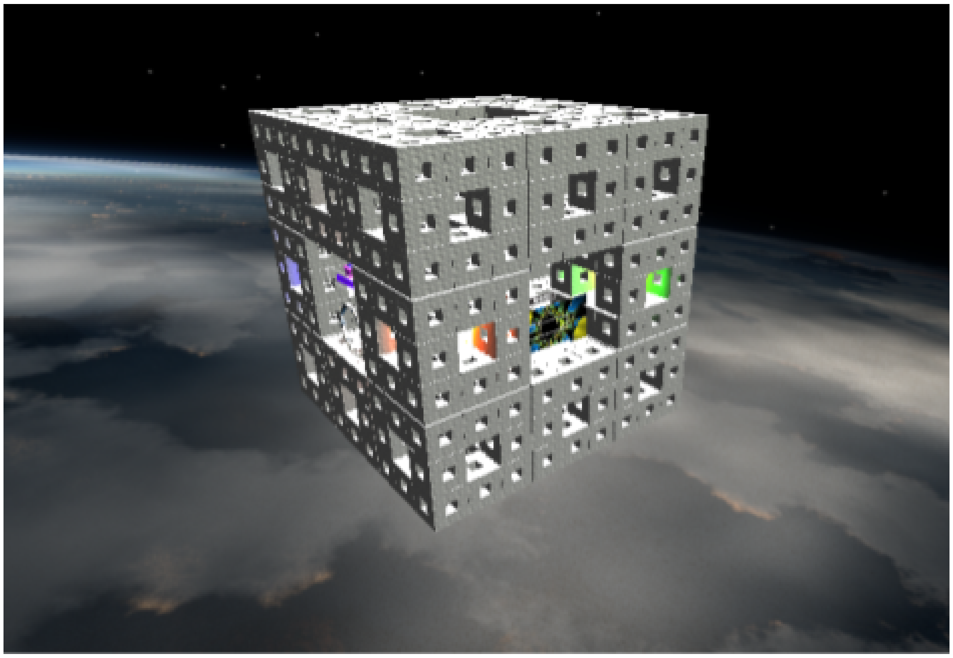


Figure 13: Menger Sponge Menu

I chose to use climbing as one of my mechanics because I thought it would give the user a better insight on how the Menger Sponge really works considering climbing is a method of exploration.

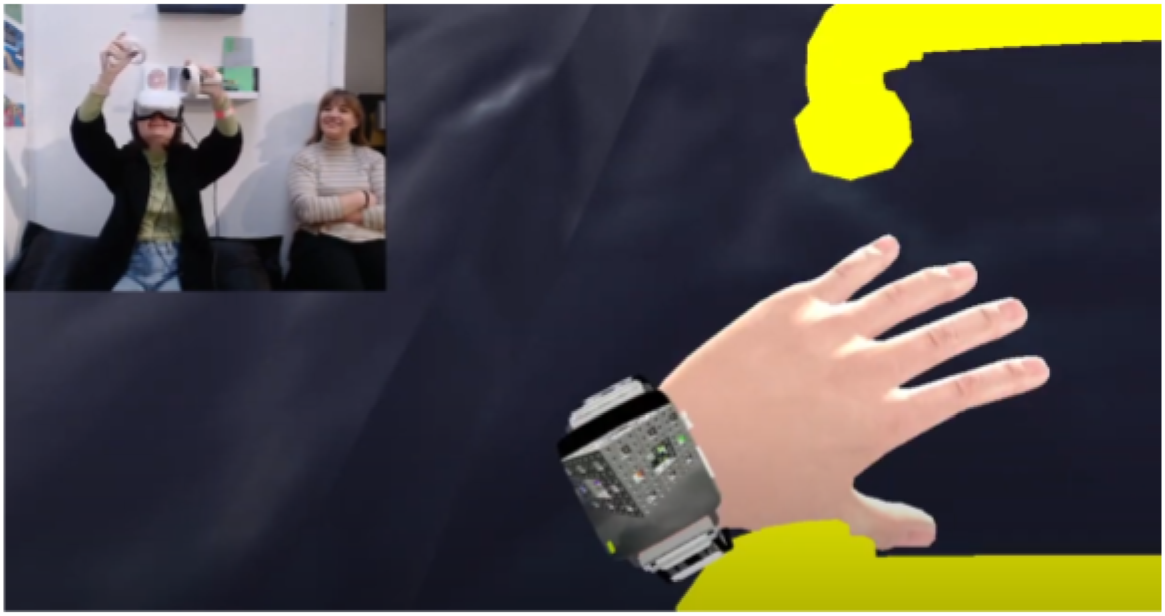


Figure 14: Chloe climbing up the walls of Menger Sponge Menu

I thought about how I wanted to form my menu and respawn UX (user experience). I thought about how in VR the player is using their hands and wanted to utilize the tracking of hands by creating a Respawn Watch.

Figure 15: Respawn watch

See Appendix L to see how the watch works.

The idea began as an exit watch that was inspired by *VRChat's* wrist menu. I used the Menger sponge picture from my hub and hoped that people would recognize it.

I added the educational bit because I wanted to show and test the learning aspect of my project and see how VR could be used as a tool.

Mandelbrot

I experimented with creating Mandelbulbs, which is the 3D evolution of the mandelbrot, my primary original focus and creation. My next step was researching and understanding 3D fractals. The mandelbrot fractal is an important mathematical concept because it demonstrates the concept of self-similarity, which is a key property of fractal geometry. The Mandelbrot set is a specific type of fractal that is generated by applying a simple mathematical formula to a complex number. This formula is repeated over and over again, resulting in a geometric shape that is infinitely complex and exhibits self-similarity at all scales.

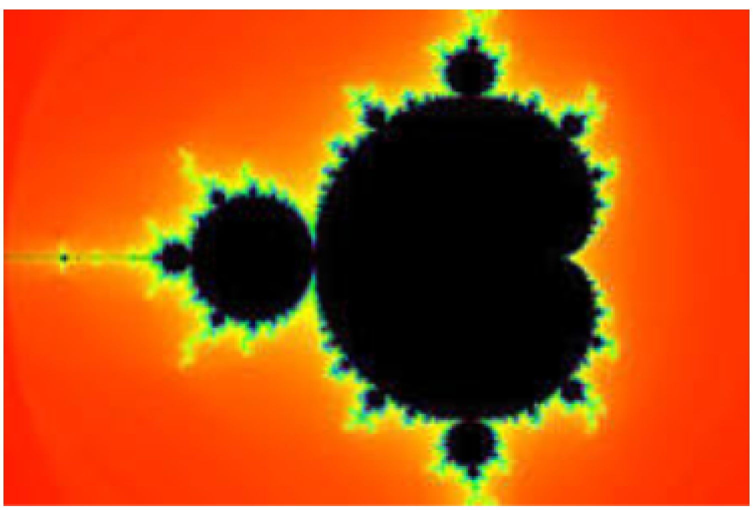


Figure 16: Mandelbrot Set

"As technology has improved, mathematically accurate, computer-drawn fractals have become more detailed. Early drawings were low-resolution black and white; later drawings were higher resolution and in color. Many examples were created by programmers working with Mandelbrot, primarily at IBM Research. These visualizations have added to the persuasiveness of the books and their impact on the scientific community." (Mandelbrot, 1983)

I started with the Mandelbrot because it was the most well known fractal and the fractal about which I found the most available research content. I began by creating this geometrical shape in unity and adjusting it to look the way I wanted. My first steps were to create a shader and successfully display the shape on a 2D plane. Next I programmed the user to be able to control where they are looking at the fractal using WASD keys, zoom in and out, and rotate the fractal with different keys. I tested if the user would be able to handle this in VR by putting someone in VR and then manually adjusting the fractal myself with my keyboard. As I suspected, the people that I tested were prone to getting sick with the rotation/spinning of the shape. The zooming in and out also caused issues with motion sickness and more significantly, I was unable to get a clear zoom for a long time as the computer could not handle zoom past specific values without getting blurry. This happened to be an issue that I ran into with my 3D fractals as well, as they could only be iterated a certain number of times before crashing Unity.
This is my original Unity and C Sharp 2D creation with zooms, movements and rotations:

See Appendix M to see my Mandedlbrot Creation 2D Zoom.

I decided to make the Mandelbrot just a skybox with 360 sound because after testing I found that the interactive mandelbrot makes people very motion sick. My next attempt was to use render textures and have this displayed around the user but this was difficult because of the scale. I liked the look of the fractals being the skybox and reflecting off of objects in my scene especially as the colors were my favorite aspects of the fractals I was using.
When developing for VR, I decided the best approach was to keep the size (zoom) of the Mandelbrot fractal controlled and give control to the user of how they could rotate the skybox. I programmed the rotation of the skybox with the same controls that they use to move and rotate themselves since I believed this would be the most intuitive and least likely to cause motion sickness.
I began with the Mandelbrot fractal because its discovery was groundbreaking. I soon realized that fractals can not only be created by using computers, but they can also be found in nature. I decided to create scenes that displayed fractals in ways the user could alter. I decided to create a main hub that held portals to different fractal scenes. The portals survey the purpose of showcasing different natural fractals, including scenes of snow, trees, shorelines and other interactive scenes. These scenes were educational, as they allowed me to teach about the

coastline paradox and show the fractals found in nature. I also added mechanics that let the user interact with the fractals, such as being able to eat a Romanesco broccoli to emphasize that fractals can be found in our everyday life.

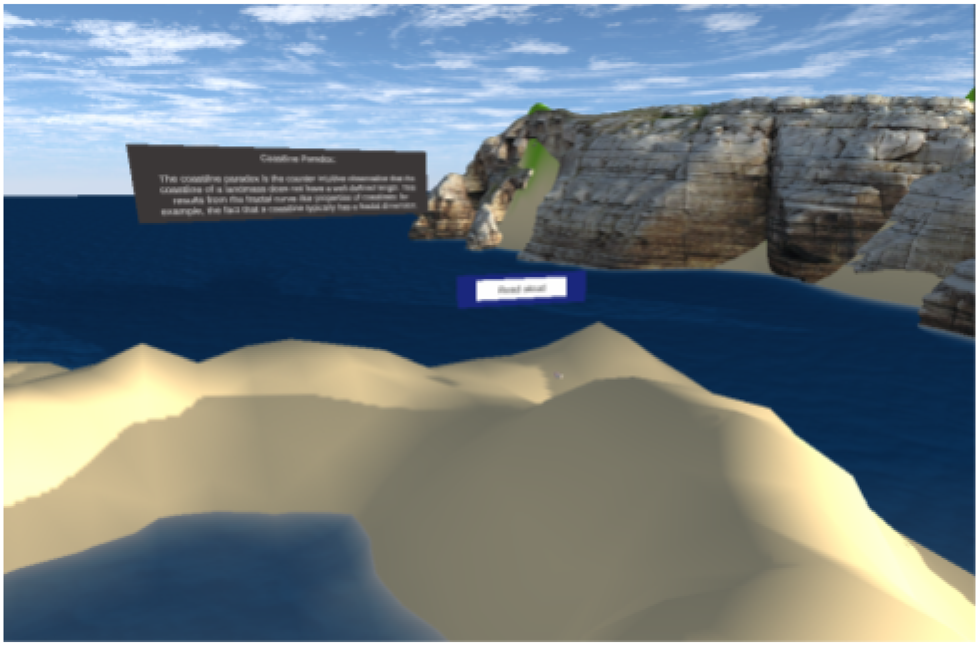


Figure 17: The Coastline Paradox, an educational scene

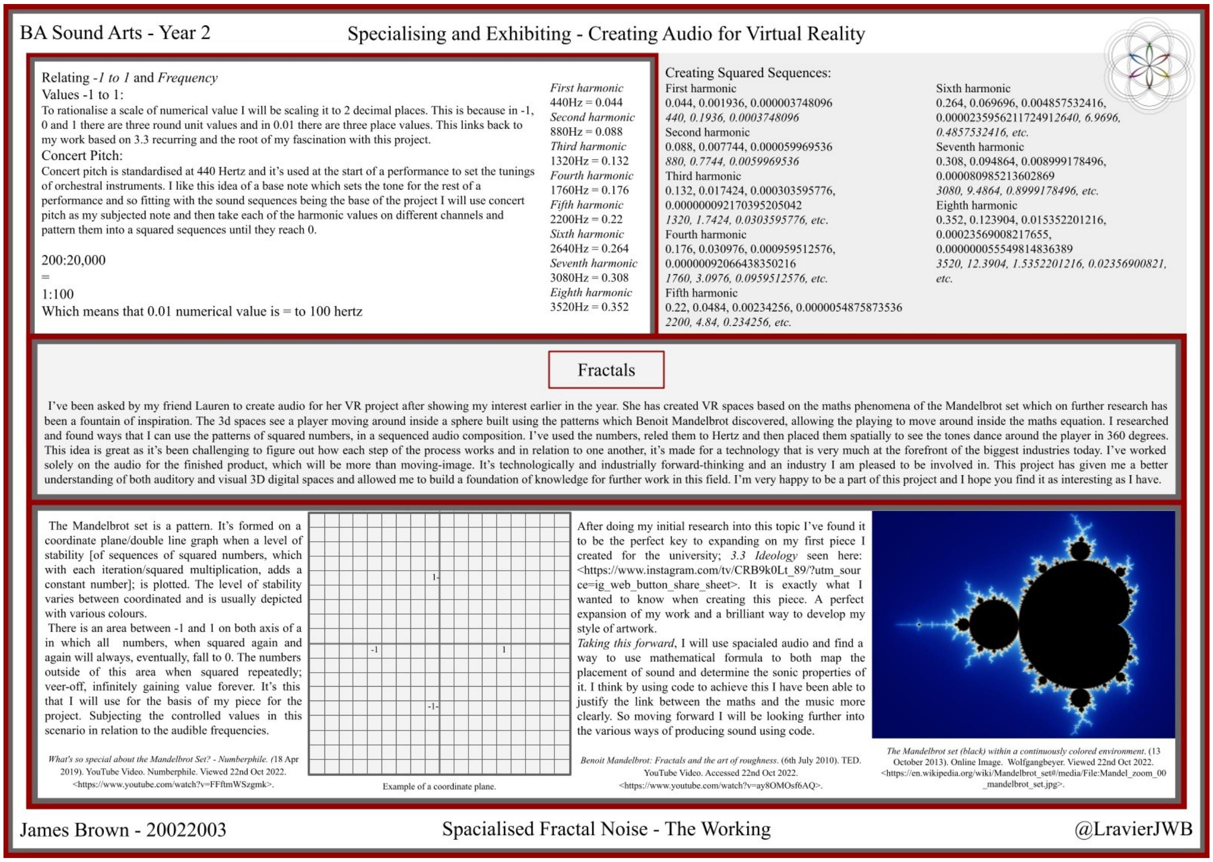
Mandelbrot sounds

The Mandelbrot fractal is a mathematical object that can be represented visually as a complex and intricate pattern. It is generated by a simple mathematical formula, but it has an infinite amount of detail and complexity.

One way that the Mandelbrot fractal can be used to generate sounds is by mapping specific aspects of the fractal, such as its shape or color, to different musical parameters. For example, the position of a point in the fractal could be mapped to the pitch of a musical note, while the color of the point could be mapped to its timbre or loudness. By using this type of mapping, it is possible to generate musical compositions that are based on the Mandelbrot fractal.

Overall, the Mandelbrot fractal is a fascinating mathematical object that can be used in a variety of interesting and creative ways, including the generation of sounds. Its infinite complexity and detail make it a powerful tool for generating complex and intricate musical compositions.

I chose this scene to work with a sound student because I knew the process and generation of this fractal before any other fractal. James worked with the Mandelbrot Set to create sounds and here is how it was done.



The user is encouraged to first enter the Mandelbrot Scene. The mandelbrot is a 2 Dimensional geometric fractal. "The Mandelbrot Set" is the most widely recognized mathematical fractal form. I also use the pythagoras fractal to introduce my users to fractals that they hopefully will recognize. Once the user is ready to move to the more complex scenes, they will need to climb to different portals. The scenes that are on the second level of the Menger Sponge are scenes with interactable skybox fractals, a natural and educational fractals scene and two mini games. Most importantly, as it is the most exciting concept of fractals to me, I would like the user to experience exploration of fractals including 3D mathematical constructs. I wanted to display the beauty and diversity of fractals while informing about them as well. I have created a fractal world with different interactions and visuals to portray the tactile magic of these concepts.

Fractals in Nature

Natural and geometric fractals (found in every fractal). A shape is self-similar when it looks essentially the same from a distance as it does closer up. Fractals occur in nature and are self-similar. If you ever looked closely at a tree and its branches and the the way a tree grows limbs this will be self evident: “The main trunk of the tree is the origin point for the Fractal and each set of branches that grow off of that main trunk subsequently have their own branches that continue to grow and have branches of their own. Eventually the branches become small enough they become twigs, and these twigs will eventually grow into bigger branches and have twigs of their own. This cycle creates an “infinite” pattern of tree branches. Each branch of the tree resembles a smaller scale version of the whole shape.” (John, 2020)

Figure 18: Fractals are found in nature such as trees

One of the key educational elements to my project is that it teaches users about the Koch snowflake iterations.

Koch Snowflake

See Appendix N for GIF of Koch Snowflake Self Iterations.

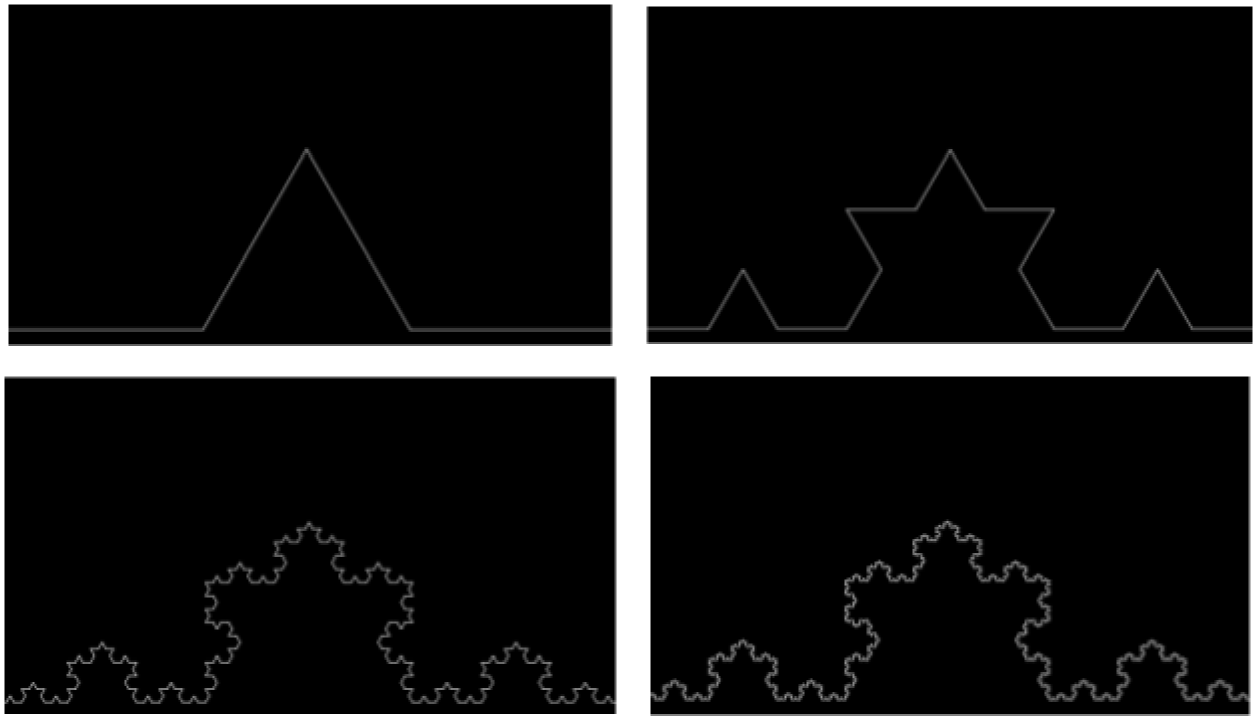


Figure 19: Koch Snowflake four iterations

The Koch snowflake is a unique fractal because it is created by starting with a triangle and then adding smaller triangles to the middle of each side. The Koch snowflake is also interesting because it is self-similar, meaning that a smaller version of the snowflake can be found within the larger one. (Processing, 2022)

I found fascination in the fact that snowflakes can be created through nature but also is a self-similar fractal. I thought this would be a good opportunity to teach my users about the Koch snowflake so I created an informational scene about the Koch snowflake and how it is created, and I also created a snowing particle effect and spawned Koch snowflakes to fall as if it was snowing. I love the snow and creating particle systems so I made a scene with snowing Koch snowflakes and added the iterations and information about how it is made and self-similar.

I wanted to make use of the virtual space that VR gives creators and put these ideas into an exploration simulation.

Pythagoras Tree

In my game I used the Pythagoras tree. Like the Menger Sponge, it is a plane fractal constructed from squares: “The construction of the Pythagoras tree begins with a square. Upon this square are constructed two squares, each scaled down by a linear factor of, such that the corners of the squares coincide pairwise. The same procedure is then applied recursively to the two smaller squares, ad infinitum.” (u/jcponce, 2022).

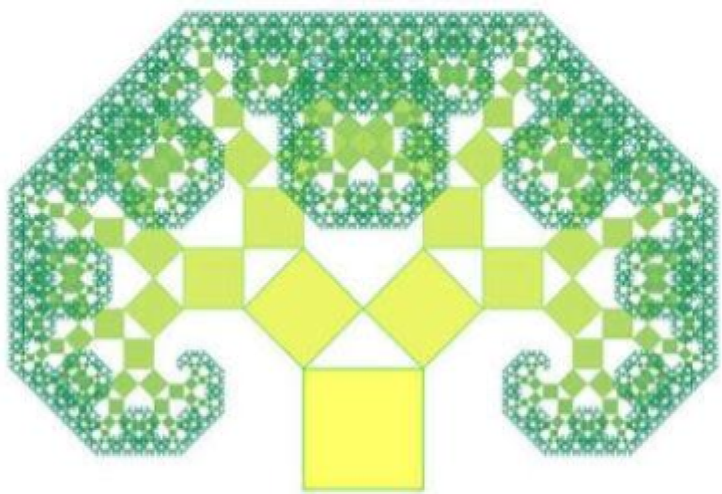


Figure 20: Pythagoras Tree

I was inspired by the golden ratio to create many various iterations of the pythagoras fractal. I created another education scene on the pythagoras fractal and pythagorean theorem. I used the golden ratio along with the fibonacci sequence to create different generations of trees within my scene. I only create pythagoras fractals up to a level of 13 iterations otherwise they would crash the computer.

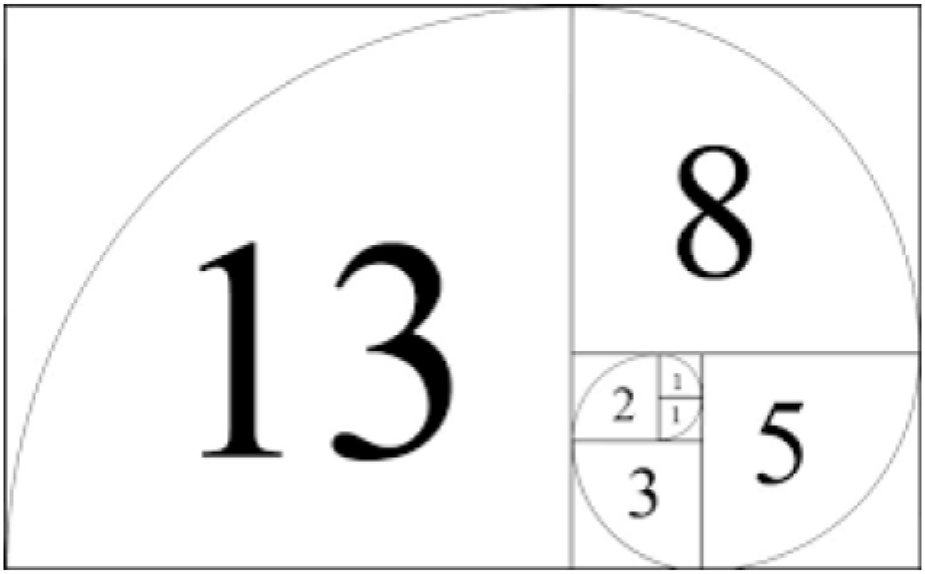


Figure 21: Golden Ratio numbers I used to create my Pythagorean Tree scene

The Golden Ratio is a mathematical concept that is often used in art, architecture and design. It exists when a line is divided into two parts and the longer part (a) divided by the smaller part (b) is equal to the sum of (a) + (b) divided by (a), which both equal 1.618. The pentagram is an example of a shape using the golden ratio. It is defined as the ratio of two numbers, where the ratio of the sum of the numbers to the larger number is the same as the ratio of the larger number to the smaller number. The golden ratio is thought to be aesthetically pleasing and has been used in a variety of contexts, including the design of buildings and artworks.

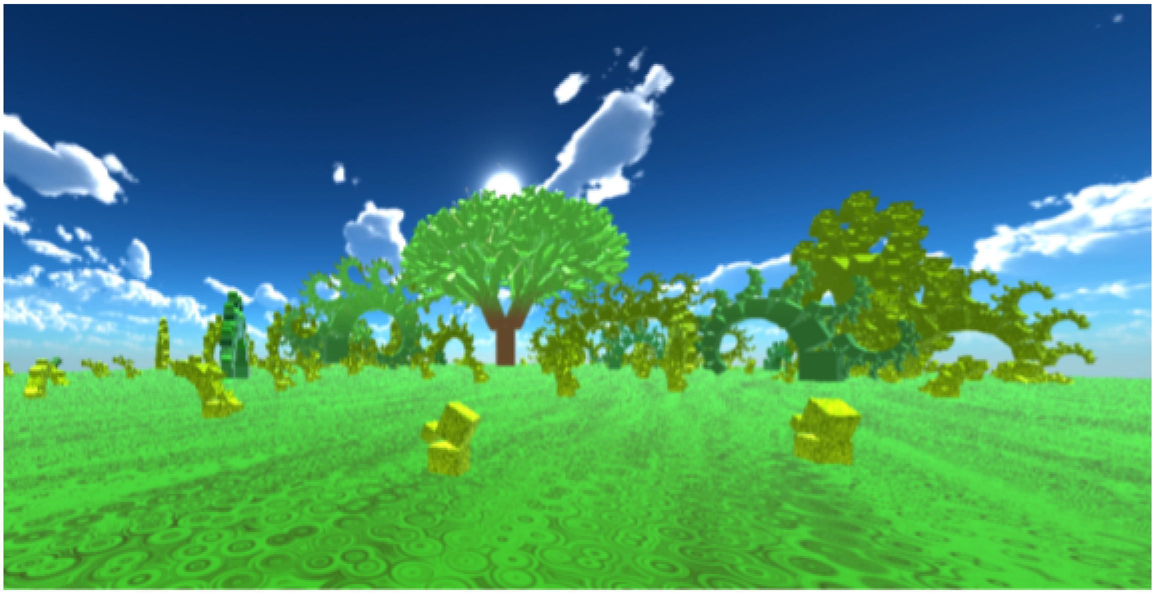


Figure 22: Pythagoras Tree fractal scene, made using golden ratio number interactions

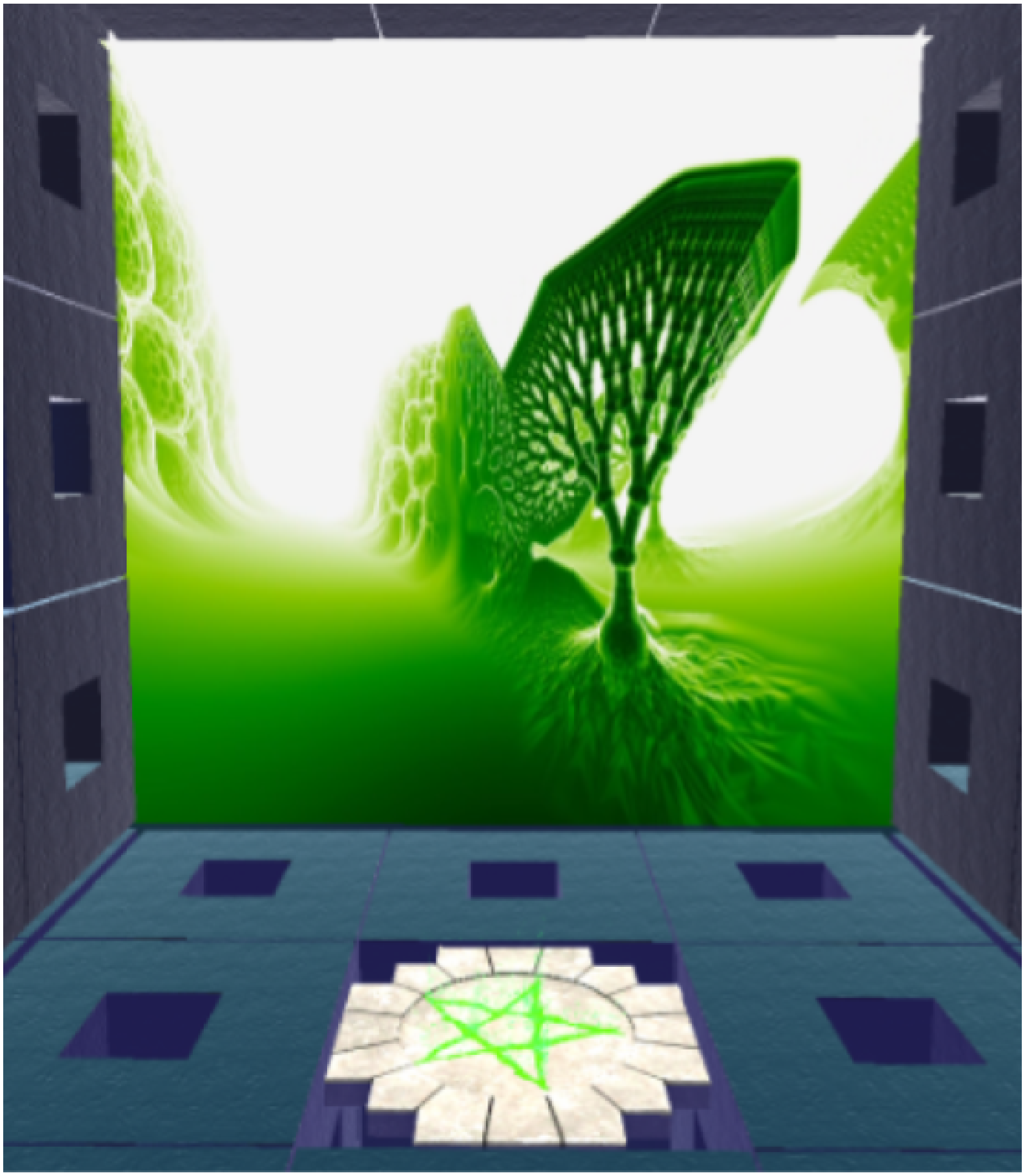


Figure 23: Pentagram portal used to teleport users into Pythagoras Tree scene

Utilizing Unity Game Engine gave me the ability to create these geometric fractals with code and shaders. I created shaders to display and immerse users inside of 3D fractals which are far more complex than being able to show and interact within a traditional game display as in displaying 3D geometry on a computer monitor or seeing a picture in a textbook.

Romanesco Broccoli

I included a Romanesco broccoli in my natural scenes menu and the user is able to pick it up and eat the real world fractal. It helps users learn about fractal geometry in a fun and realistic way by referencing a real life object and pairing it through game mechanics to a familiar item.

"The Romanesco broccoli is a big spiral covered in little spirals that are covered in even more little spirals, and so on until the plant cells cannot construct a spiral small enough. This is an example of a fractal occurring in nature. Self-similarity fractal." (Lumenlearning.com, 2022)

Figure 24: Romanesco broccoli fractal

To accurately comprehend my project, one must appreciate Chaos Theory. Chaos Theory is a branch of math that studies the behavior of dynamic systems that are sensitive to initial conditions, often referred to as the butterfly effect. "Chaos is the science of surprises, of the nonlinear and the unpredictable." (Fractalfoundation.org, 2018). I chose to incorporate chaos theory into my project during early testing when I heard a lot of people describe the concepts as being very random. I believe this came from the idea of the world being all over the place. I figured that since Hypperbolica made it work, so could I.

Chaos theory is commonly used in the study of weather patterns, which can exhibit erratic behavior. I used chaos theory to justify my randomness in interaction in my scenes. Chaos theory is demonstrable in my project through randomness such as the pentagram which is a spawn for my portals. One way that chaos theory can be incorporated into games is by randomness and unpredictable events. I thought this would pair well with my abstract concepts and make the game more dynamic and engaging for the player, as they will never know what to expect. Eventually I would like to evolve those concepts into non-euclidean portals as I saw in *Hyperbolica*. I wanted to create an illusion that mesmerized the participants. I believe that I successfully did since I had people engaging in Featuring: Fractals for up to 30-45 minutes. I created a project where I wanted to show the capabilities of VR. I wanted to show how VR could be used as a tool - for education, for mathematical programs, for exploration, for 3D spaces, for labyrinths/mazes.

Chapter 4. Applied Methodologies of Research

Through survey participation, user testing was conducted before and after the experience. Initially, pre-experience surveys were undertaken to get an idea of what people knew about fractals and what they wanted to experience; however, most of my testing was done at the degree show once my project was complete. I found this beneficial because I had a project design for which I was satisfied and I could gather opinions for future works. I wanted clear feedback on the project and most importantly, I could accurately ascertain potential outcomes achieved from my user's point of view.
See Appendix O for User Before and After Testing Surveys.

Pre testing

I think information from this specific question might contain selection bias as many of the responders had exposure to the concept through my previous presentations. If I were to conduct the survey again, it would be useful to have a more diverse and randomly selected group of participants. This should provide more reliable results.

<u>Question</u>

"What do you think are some characteristics of fractals?"

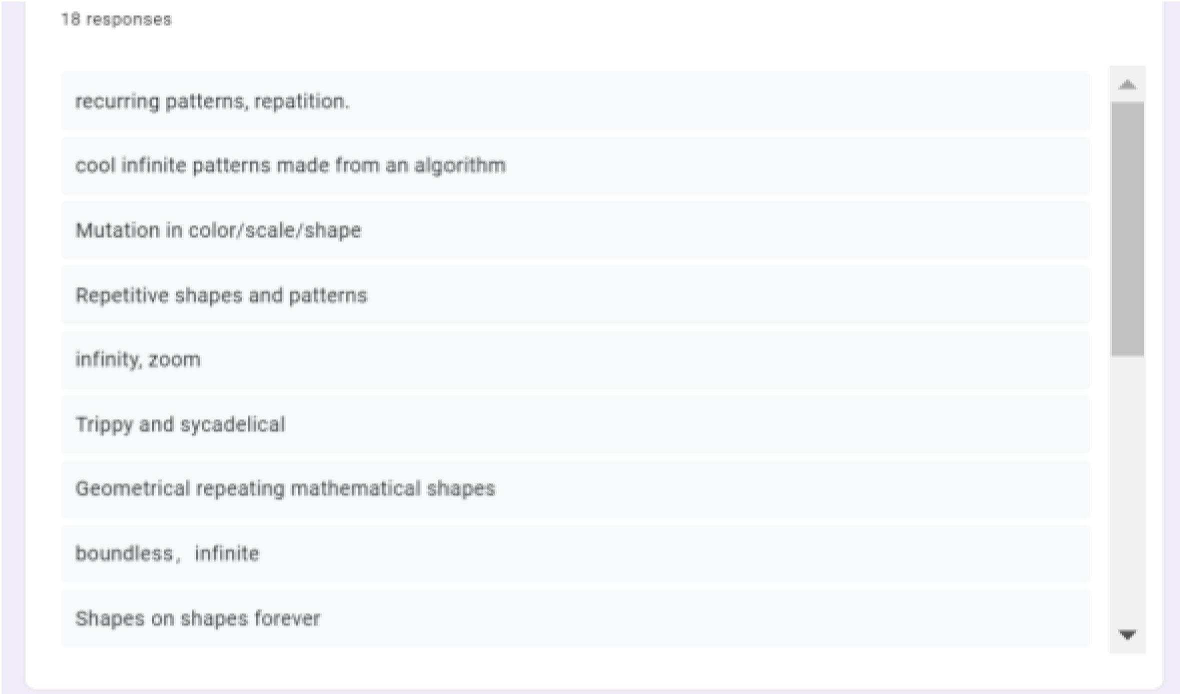


Figure 25: Responses to Question “What do you think are some characteristics of fractals?”

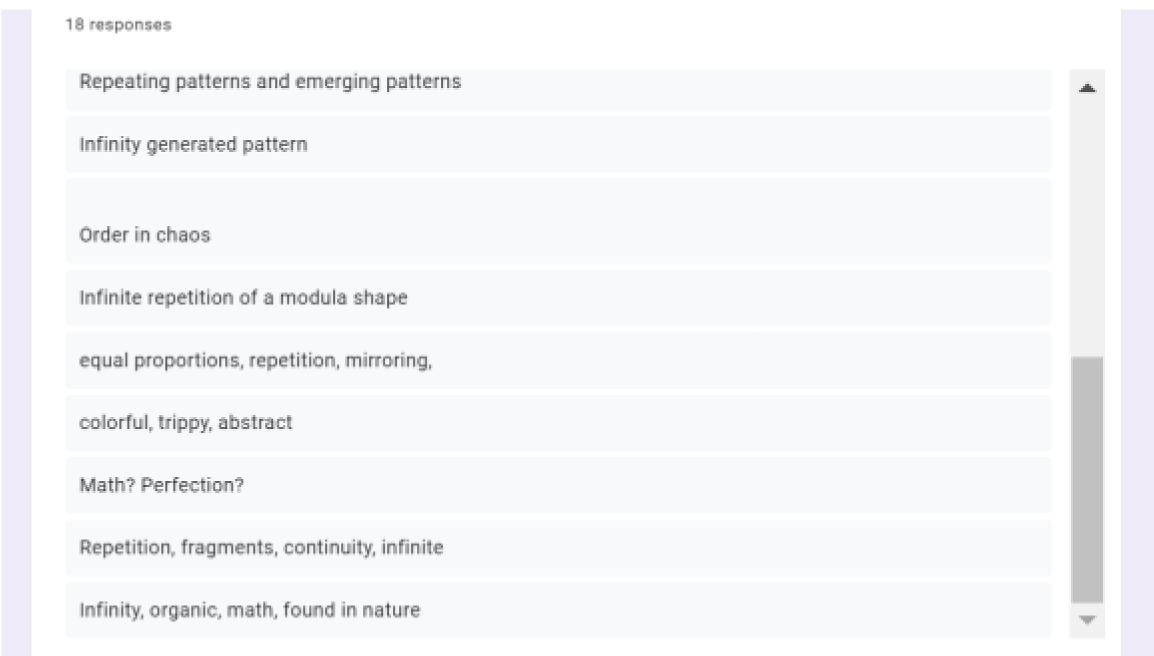


Figure 26: Through this testing, results resembled infinite and chaotic concepts

Question: “What would you like to see and/or learn about fractals?”

This Question inspired me to branch out from the mandelbrot set and natural fractals. The answers that were most provocative included a “world generated in this style?”, “Can they be 3D shapes or only 2D”, and “What other ones exist outside of the Mandelbrot set.” These answers led me to explore other fractals unknown to me and thus I found Fractal Generator, a unity package that I used to create 3D fractals: the Koch Snowflake, Pythagoras Tree, and the Menger Sponge.

Question: “Have you experienced something in VR that would not be possible without this medium? If so, what is it?”

Once again, my primary goal was to create something unique in my project that would not be possible without this medium. This question strives to pick my users’ brain and see what they think cannot be created in real life but is possible with immersive technology. The main responses that stuck out to me were “Being able connect with objects, shapes and spaces in a personal manner” and “Feeling of falling without moving”

In my game, I decided to use the Menger Sponge as the main hub because it fit the abstract mystery I intended to create. I placed two portals on the level where the player spawns so they could be introduced to more simple fractal scenes. I also put the portals leading to interactive fractal scenes inside one of the Menger Sponge hole iterations. To draw the users’ attention to these portals, I used lights to illuminate this area and I created yellow climbing rings designed to ensnare the user and prompt them to climb to the second level of the Menger Sponge. Additionally, I thought incorporating climbing as a mechanism would further immerse the player and make them feel connected to the space. This decision was solidified by feedback from one of my users, who mentioned that you feel like you are falling in VR even if you’re not actually moving. My goal is to fully immerse the player in the experience that I have created.

Post Testing

I found that testing the users before and after they went through my experience was beneficial because this gave me direct feedback on their encounter. The primary concern the responders noted was motion sickness.

Survey responses provided essential feedback that was used to improve my platform. My testers suggested two movement mechanics that I believe enhanced my final product as simplicity in the core mechanics made sense, noting that fractals are already trippy enough. I would change the turning method to snap turning and I would also allow the users to teleport to the second level instead of requiring them to climb the ladder every time which some people found too difficult, tiring or unnecessary. Aside from motion sickness, the other issue I heard repeated was my UI. I believe that a solution to my UI being more clear.

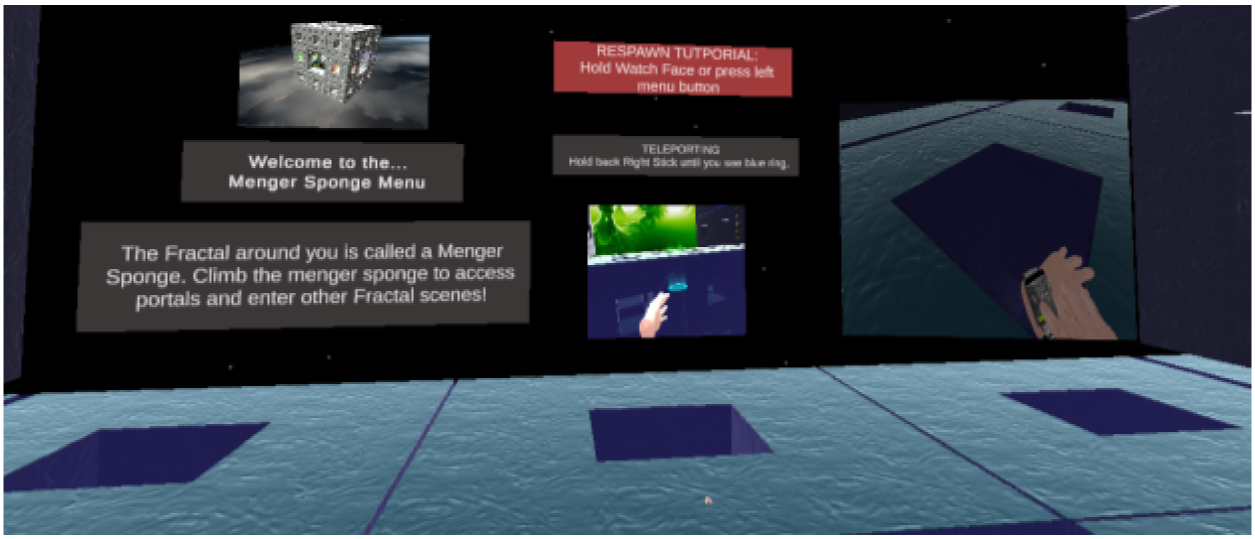


Figure 27: User interface on Menger Sponge Menu

In order to improve the user interface, a tutorial scene was added. This tutorial introduces the user to the text and information at first spawn inside of the Menger Sponge Menu space.

Question: “What did you learn from this experience?”

One answer received from this question was “That you don’t have an understanding of time in VR.” This gave me a feeling of accomplishment as I wanted to create an exploratory illusion where users could relax and enjoy without feeling connected to reality. Responses to my experience like this was exactly the illusion that I wanted to create for my user.

One of my main goals was to create an interaction that was unique to VR. I wanted to emphasize how important user hand and head tracking can be. This is why I chose climbing and archery along with other unique methods of interaction specific to 360 spaces.

I used the following question to see how I could effectively use my player's perspective of VR and interactive mechanics:"What about being in VR made this experience special that you could not get from a desktop or traditional platform?"

I knew I successfully achieved my goal from the feedback I got such as "This was like a VR world built upon math and stepping into a world that borders on your own world." Seeing this feedback made me feel satisfied as I could see that the participants were immersed in the experience. I believe the power of 360 interactive space is extraordinary and creates collaborative opportunities for game designers and artists.

I received one intriguing answer from someone saying that they got to "ride a fractal". After thinking about that user review, I changed the scene name to Fractal Roller Coaster. The feedback provided taught me the real value of VR. Virtual Reality is a multipurpose tool that has many new and spatial ways to display content creation and art especially giving the user a feeling of motion without actually moving them at all.

Finally, the main achievement I wanted to get out of this testing was to see if my users learned anything. To do so, I asked these questions: "What about being in VR made this experience special that you could not get from a desktop or traditional platform?" and I received this answer "Immersive learning keeps you focused on one main thing while on a desktop you might get distracted easily."

VR has the potential to lower the odds of getting distracted and I was happy to see that this showed in my project especially considering the fact that most of the testing was done in a very busy space. VR immerses the user fully in a digital environment and I wanted my user to separate further and further from the real world. To do this, I included engaging and interactive worlds that got the user moving at times but also had them just watching and listening at others. Based upon testing results I have concluded that VR can be used as an effective learning tool for education and retention. Answers such as the following prove this assumption:

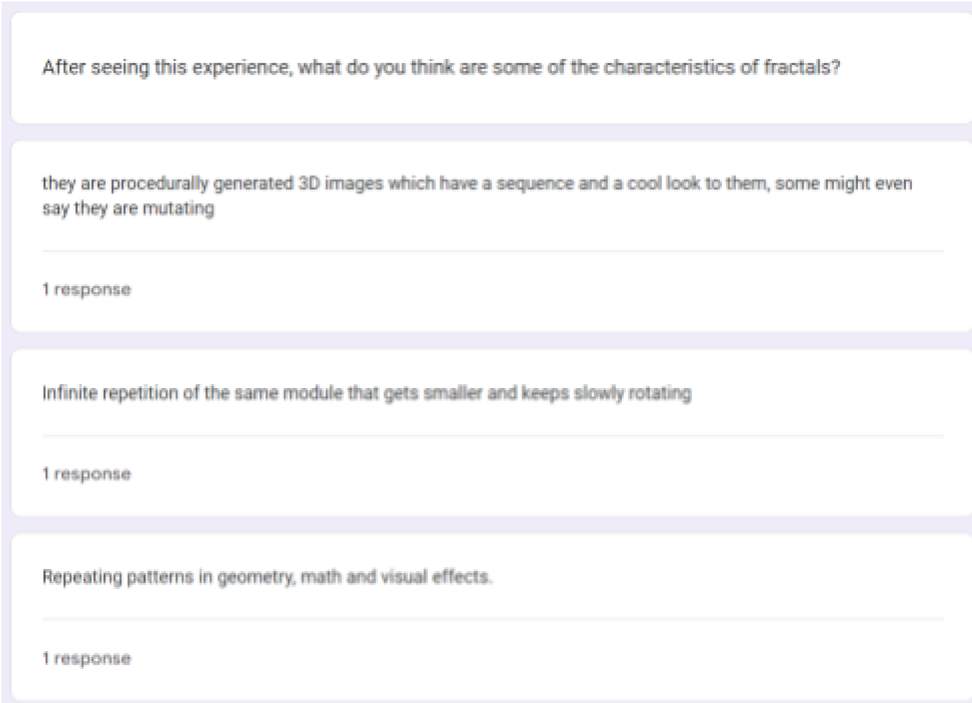


Figure 28: Responses from user texting

Overall, I have concluded that VR has the potential to positively impact the field of education by providing highly immersive and engaging learning experiences with minimal distractions. My project shows that VR allows people to explore new places in concepts while having fun with gamification and visual effects.

Chapter 5. Critical Analysis

Issues

Motion sickness is a common issue in VR Games, and it can be a major obstacle for players who want to enjoy the immersive experience. I found that motion sickness is amplified when working with fractals which can already be nauseating to some people even viewing in 2D. In my VR game, I worked on addressing this issue by implementing verbal and written warnings to the user, highlighting interactions that were smooth, and testing to see what causes nausea in my users.

One of the main challenges I faced early on when creating my first fractal, the Mandelbrot set, was the potential for motion sickness when incorporating interaction such as zooming and spinning the fractal as a skybox. With research and testing the best option was to allow users to move the skybox around in the same way they would move their XR Rig. This way, they could explore the fractal at their own pace and avoid any potential issues with motion sickness.

A major trigger of virtual reality sickness is when there is disparity in apparent motion between the visual and vestibular stimuli. This disparity occurs if there is a disagreement between what the stimuli from the eyes and inner ear are sending to the brain. (Wikipedia Contributors, 2022). When your brain thinks you are moving, but your body is static, it creates a disconnect between the two that causes enough confusion to make you feel ill. (Coles, 2021). Due to the nature of my game, I found challenges in my Fractal Rollercoaster scene attributable to the nature of the shader. Since I was using the shader as a skybox, I could not control the fact that the player's head was moving horizontally bouncing from one side of the installation to the other. I assumed that I could adjust the camera movement inside of the shader but since I converted the shader from Shadertoy(GLSL) to Unity(HLSL) I was not familiar with adjusting the camera movement and how it was portrayed. I noticed the reactions to this fractal rollercoaster scene were widely variable as some people hated it and others couldn't get enough (for example, one player got inside of this scene, screamed and then immediately came out, whilst others sank into the beanbag and watched this scene of animated fractals for more than 5-10 minutes).

To mitigate the impact of motion sickness that was occuring in some of my users, I offered different levels of interaction while going through my experience (seated in a chair, seated on a bean bag, and standing). I found that a combination of all of these methods optimized the experience because that offered the user decompression between scenes and some mechanics and scenes were better suited for different positions such as: the climbing mechanic was well suited for standing, and Fractal Roller Coaster was best suited for the bean bags.

Limitations

Time and motion sickness were the limitations I encountered. Motion sickness was experienced by some of my users. Some scenes, such as those using moving shaders as a skybox, were the most sickening to my users. It was likely that the user would not continue their experience after these two particular scenes: Fractal Rollercoaster and Mandelbrot set. I believe this occurs because conceptually the world is moving around them while they are in a stationary position.

Other issues were not as significant but did help me narrow down the concept I was using to create my experience. In the beginning I wanted to create non-euclidean portals but because of Virtual Reality rendering two cameras, I tried to make non-euclidean portals but they did not work well in VR. This led me to explore other mathematical equations that could be displayed and leveraged within VR. I came across fractal geometry and decided that I would use fractals to create my experience.

Future Work

I would like to recreate the mandelbrot set as an Augmented Reality application. I enjoyed adding the zooming and rotating in Unity to my shader, however; this was the most likely to cause nausea so I decided to take out this functionality and just have the mandelbrot shader viewable and for the user to enjoy to look at but not interact with zooming in and out and rotating the fractal. I found that after creating the mandelbrot fractal, this fractal was a bit of a disadvantage in terms of displaying it in VR. I had no way to make this 2D fractal visible except with projections and in skyboxes.

I decided to research the mandelbulb fractal and whilst doing this I discovered other fractals that were displayed as three dimensional fractals. I found a package called Fractal generator and began to create scenes with 3D fractals that I generated.

Chapter 6. Conclusion

Overall, the use of fractals in VR has the potential to greatly enhance the realism and complexity of virtual environments, and has a wide range of applications in fields such as entertainment, education, and training.

My goal was to create a psychedelic, illusionary experience and I achieved that especially in Fractal RollerCoaster by using alternative methods of experience such as sitting in a chair, bean bag or standing. The bean bags provided a sense of relaxation and comfort especially as Fractal RollerCoast had an infinitely animated fractal shader. Furthermore, to deepen the sense of relaxation, I added to the effect by including Theta waves and as a result the majority of the respondents spent significantly more time immersed in the scene compared to when there was no sound or bean bag.

Fractals are complex geometric patterns that can be explored in virtual reality. Virtual reality allows for the exploration of these patterns in a way that is not possible in the physical world, as it allows for the manipulation of the fractal that is not possible with traditional methods. This allows for a deeper understanding and appreciation of the beauty and complexity of fractals. Additionally, virtual reality can be used to create interactive experiences that allow users to explore and manipulate fractals in a more intuitive and immersive way. Overall, the use of virtual reality technology has the potential to greatly enhance our understanding and appreciation of fractals.

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Appendixes

Appendix A

Fractals Multiverses in VR, 2018

Source

Generation of 3D Fractal Images for Mandelbrot and Julia Sets, 2022

Source

Educational Virtual Environments: A ten-year review of empirical research(1999–2009)

Source

Appendix B

YouTube Sky Full of Fractals Gameplay

Youtube Gameplay Despoina Archery Scene and Walkthrough

Appendix C

Museum Of Other Realities Gameplay

Appendix D

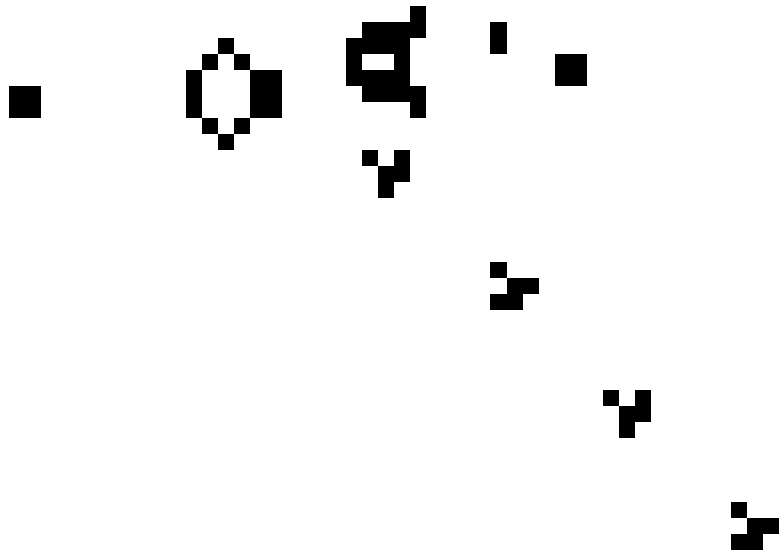
World: ADRIFT World

Created by: M E R C

World: Musical Flowers
Created by: Luftprut
Fractal World! VRChat Gameplay

Appendix E

Conway's Game of Life



"The universe of the Game of Life is an infinite, two-dimensional orthogonal grid of square cells, each of which is in one of two possible states, live or dead (or populated and unpopulated, respectively). Every cell interacts with its eight neighbours, which are the cells that are horizontally, vertically, or diagonally adjacent. At each step in time, the following transitions occur:

1. Any live cell with fewer than two live neighbours dies, as if by underpopulation.
2. Any live cell with two or three live neighbours lives on to the next generation.
3. Any live cell with more than three live neighbours dies, as if by overpopulation.
4. Any dead cell with exactly three live neighbours becomes a live cell, as if by reproduction.

These rules, which compare the behaviour of the automaton to real life, can be condensed into the following:

1. Any live cell with two or three live neighbours survives.
2. Any dead cell with three live neighbours becomes a live cell.
3. All other live cells die in the next generation. Similarly, all other dead cells stay dead." (Wikipedia Contributors, 2022)

Appendix F

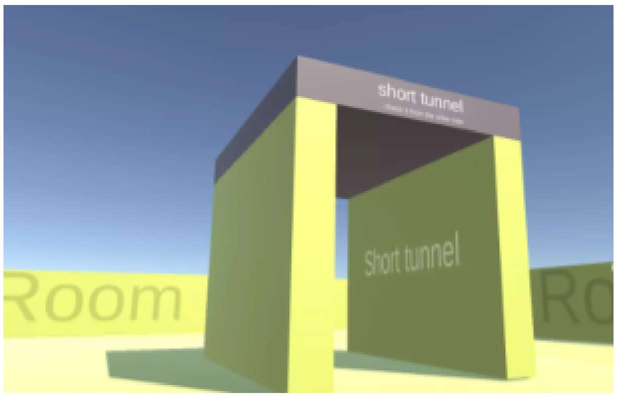
Hyperbolica game

Hyperbolica

Interview with Hyperbolica Game Developer

Appendix G

Non-Euclidean Portals



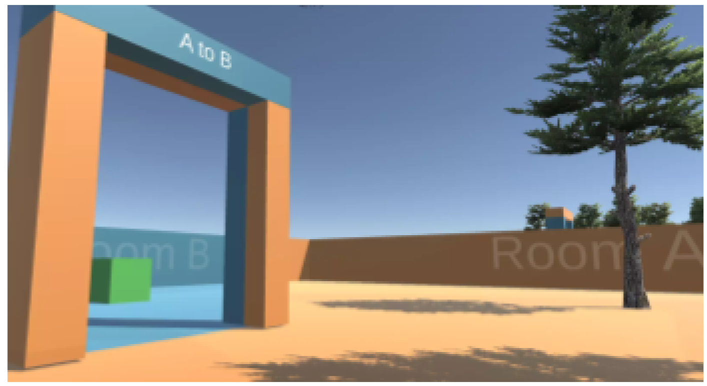


Figure 29: Non-euclidean portals

Appendix H

Testing Part 1

Appendix I

Hurricane VR - Physics Interaction Toolkit

Appendix J

Menger Sponge Animations

Appendix K

Fractal Generator

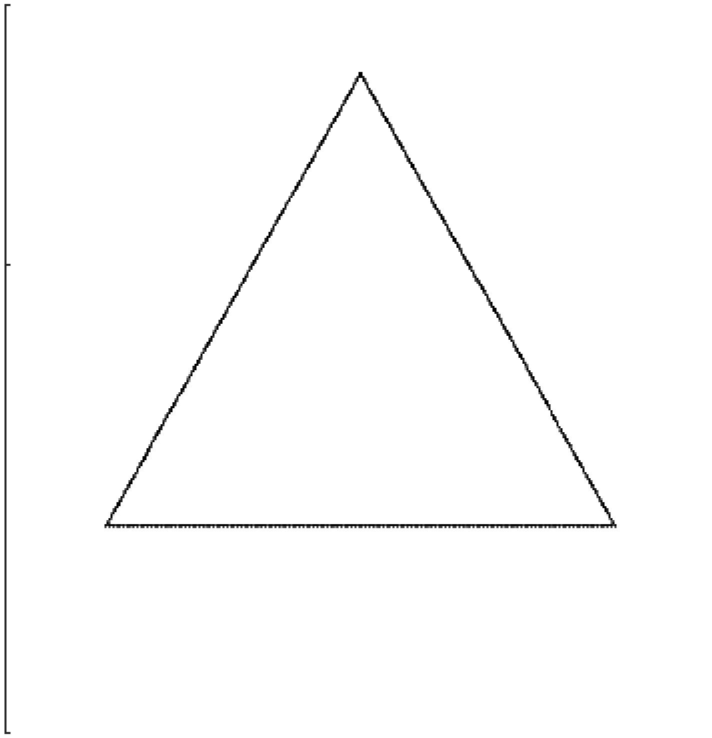
Appendix L

Watch Demo

Appendix M

Mandelbrot 2D zoom Lauren

Appendix N



Appendix O

Pre VR User Testing

Post VR User Testing